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Drift functions and non-explosiveness of countable state Markov processes

Abstract

We consider a Markov process X on a countable state space E with transition operator $\{P_t\}_{t\geq 0}$. We assume that X is a right-continuous, standard, stable, and conservative. It is well-known that then the q-matrix Q exists, and that the Kolmogorov forward and backward equations hold elementwise.

Let $c \in \mathbb{R}$. The function $V : E \to (0, \infty)$ is called a *c-drift function for Q*, if $QV \leq cV$; it is called a *moment function*, if there exists an increasing sequence $\{K_n\}_n \subset E$, converging to E, such that $\lim_{n\to\infty} \inf_{x \notin K_n} V(x) \to \infty$.

It is an old result by Mu Fa Chen that the existence of a c-drift moment function for some constant $c \in \mathbb{R}$ implies non-explosiveness of the Markov process. We have proved recently that the reverse also holds true: if the process is non-explosive, then there exists a c-drift function moment V for some constant c.

c-drift functions play an important role as well, when considering rewards associated with each state of the Markov process. Using a transformation associated with a c-drift function V allows to determine a class of reward functions $f : E \to \mathbb{R}$ for which the process $M_t = f(X_t) - f(X_0) + \int_0^t Qf(X_s)ds$, $t \geq 0$, is a martingale: essentially the transformed process should be non-explosive, and then functions f bounded by V have the desired property.

c-drift functions play a similar role, when studying continuity properties of $a \mapsto \mathsf{E}_x f(X_t(a))$, where now $\{X(a)\}_{a\in A}$ is a parametrised collection of Markov processes and A is a parameter set.